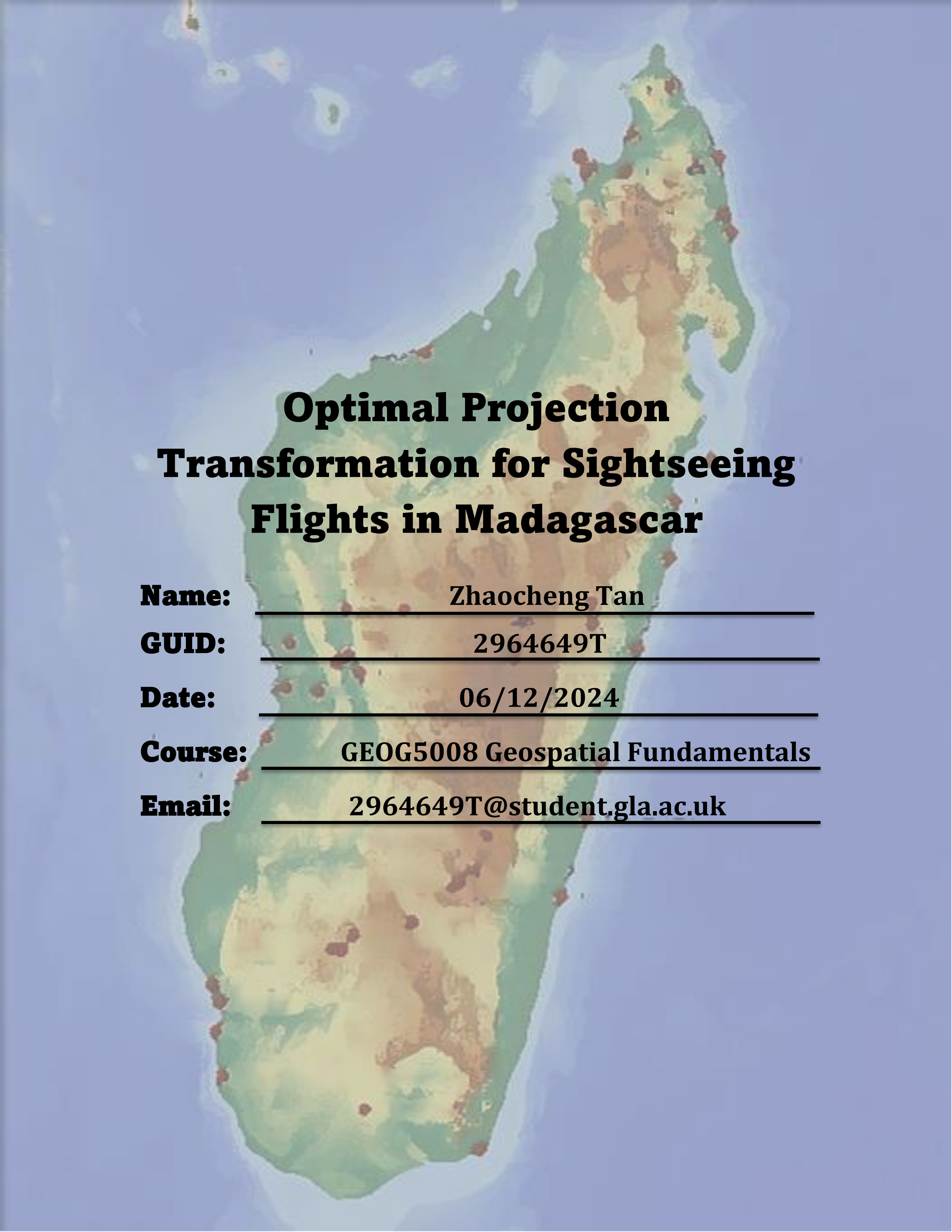

Optimal Projection Transformation for Sightseeing Flights in Madagascar

Name: Zhaocheng Tan

GUID: 2964649T

Date: 06/12/2024

Course: GEOG5008 Geospatial Fundamentals

Email: 2964649T@student.gla.ac.uk

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Abstract

This report investigates the transformation of a Mercator projection to a Lambert Conformal Conic (LCC) projection for Madagascar. The LCC projection reduces distortions and improves regional mapping accuracy. Python-based computational workflows are employed, guided by theoretical principles from Snyder's authoritative work on map projections. The report also evaluates the impact of Least Squares Adjustment on transformation accuracy and assesses cost savings for aviation routes.

1. Introduction

Madagascar, the fourth-largest island in the world, is located in the Indian Ocean. It has a central plateau, steep eastern cliffs, and lower plains with river deltas to the west. Situated near the Tropic of Capricorn at higher latitudes, the island causes significant distortion in area and distance when using the Mercator projection, making regional mapping less accurate. Therefore, projections better suited for mid-latitude regions, like the LCC projection, are more appropriate. Map projections are essential in geospatial science for representing a spherical Earth on a flat surface. However, they introduce distortions in size, shape, and distance, requiring careful selection for specific uses. This study switches Madagascar's map from Mercator to LCC projection to aid Lillybank Tours' flight planning, using Snyder's framework and Python for an accurate transformation.

Figure 1 Map of Madagascar

2. Background

2.1 Mercator Projection: Properties and Challenges

The Mercator projection, known for preserving angles, is useful for flight route planning, especially for calculating route directions. It works well for shorter coastal routes within Madagascar, as it accurately maintains angles. However, due to the island's proximity to the Tropic of Capricorn, particularly in the southern region around 25°S, the Mercator projection distorts area and distance at higher latitudes. While Madagascar doesn't extend into regions above 40°S, the southern parts still experience noticeable distortion. For shorter island routes, this has little impact, but for longer routes, such as those between the north (e.g., Antananarivo) and the south (e.g., Toliara), the distortion can affect accuracy. Therefore, for longer routes, the LCC projection is more suitable, as it reduces distortion in higher latitudes and provides more accurate mapping.

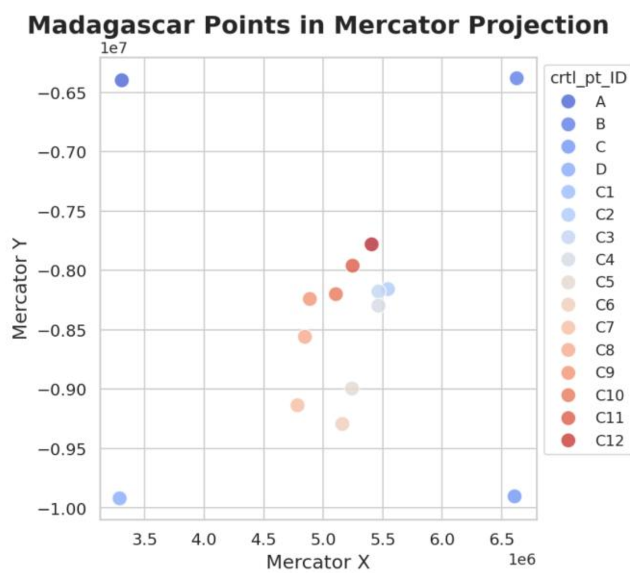

Figure 2 Madagascar Points in Mercator Projection

2.2 Lambert Conformal Conic Projection: Features and Applications

LCC projection is effective for preserving shape and minimizing distortion in mid-latitude regions. By aligning its standard parallels with Madagascar's latitude range (10°S to 25°S), the LCC ensures high accuracy for flight route planning. Within this range, it reduces area and distance distortion, improving the precision of flight distance and angle calculations, especially for intra- and inter-island flights. However, the LCC projection has limitations. Its accuracy depends on the choice of standard parallels. For routes with a large latitude difference, such as from Antananarivo (north) to Toliara (south), distortion can be noticeable. Therefore, while LCC is ideal for most routes within Madagascar, longer routes or those crossing significant latitude differences may require adjustments or alternative projections.

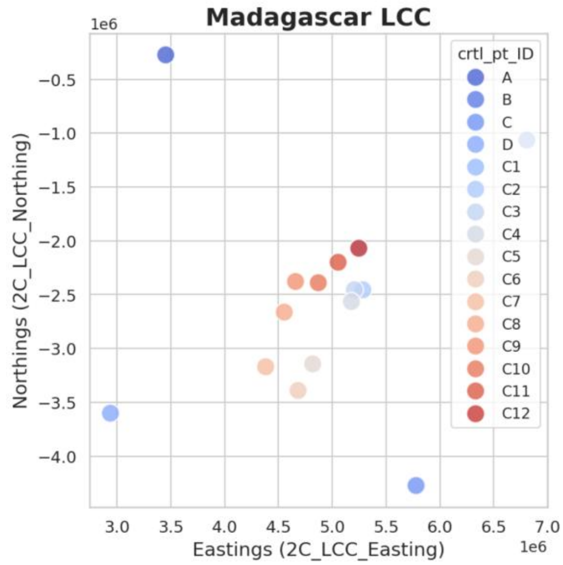

Figure 3 Madagascar Points in LCC Projection

3. Methodology

3.1 Mathematical Foundations

1) Mercator to Geographic Coordinates:

The formula to convert from Mercator coordinates (x, y) to geographic coordinates (latitude φ and longitude λ) is:

- **Latitude**:

$$\phi = 2\arctan(e^{\frac{y}{R}}) - \frac{\pi}{2}$$

- **Longitude**:

$$\lambda = \frac{x}{R}$$

Where:

- **R** is the Earth's radius.
- **x** and **y** are the Mercator projection coordinates.
- **e** is the base of the natural logarithm.

2) 2. Geographic to LCC Projection:

To transform geographic coordinates (latitude φ and longitude λ) to LCC projection coordinates (X, Y), we use the following equations:

- **X coordinate**:

$$X = \rho \sin(\theta)$$

- **Y coordinate**:

$$Y = \rho_0 - \rho \cos(\theta)$$

Where:

- **ρ** is the radial distance from the origin.
- **θ** is the longitude (λ) in radians.
- $\boldsymbol{\rho_0}$ is the reference latitude.
- **n, F,** $\boldsymbol{\rho_0}$ are constants derived from the standard parallels (the latitudes where the projection is most accurate).

3.2 Workflow Design in Python

The Python script implements the following steps:

Figure 4 Project workflow diagram

(1) Data Input:

Import digitized coastline points and validate the data to ensure proper formatting and accuracy.

	crtl_pt_ID	Lat_deg_N	Long_deg_E	pixel_x	pixel_y
0	A	0.0	30.0	419	163
1	B	0.0	60.0	586	163
2	C	-30.0	60.0	586	340
3	D	-30.0	30.0	419	340
4	C1	NaN	NaN	525	233
5	C2	NaN	NaN	532	252
6	C3	NaN	NaN	528	253
7	C4	NaN	NaN	528	259
8	C5	NaN	NaN	517	294
9	C6	NaN	NaN	513	309
10	C7	NaN	NaN	494	301
11	C8	NaN	NaN	497	272
12	C9	NaN	NaN	499	256
13	C10	NaN	NaN	510	254
14	C11	NaN	NaN	517	242
15	C12	NaN	NaN	525	233

Figure 5 Data Input

(2) Step-by-Step Transformation:

The transformation process is broken into clearly defined steps in the Python script:

- **Geographic to Mercator Projection**:

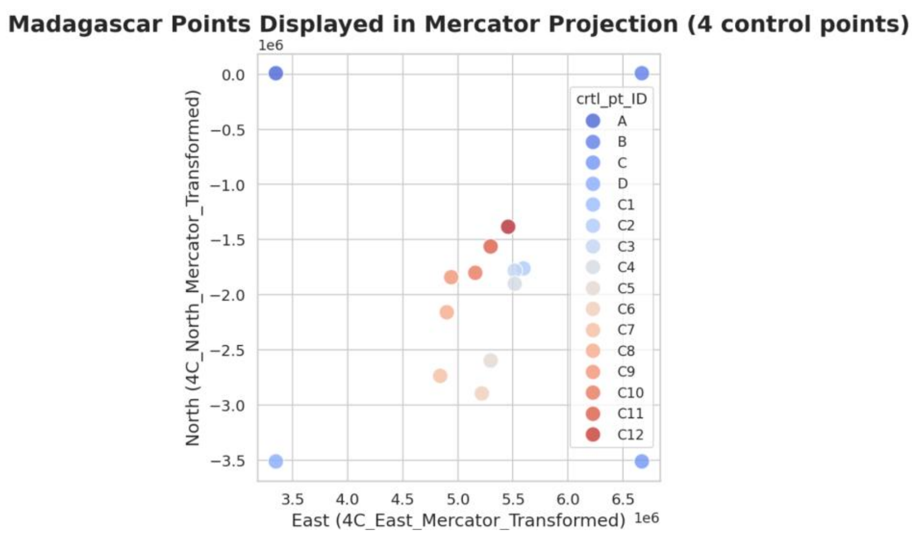

Figure6 Madagascar Points Displayed in Mercator Projection (4 control points)

- **Geographic to LCC Projection**:

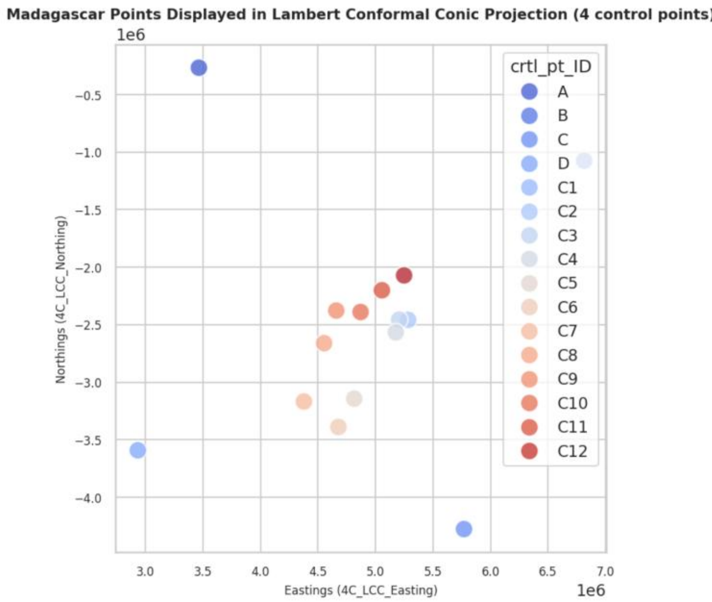

Figure 7 Madagascar Points Displayed in LCC Projection (4 control points)

(3) *Validation:*

- **Comparison with Control Points**:

The script compares the transformed coordinates with known control points by calculating the Euclidean distance between each transformed point and its corresponding control point. The differences are calculated for both geographic coordinates (latitude and longitude) and LCC projection coordinates (X, Y), helping to identify any discrepancies between the transformed and actual coordinates.

- **Error Calculation**:

The code calculates the errors in latitude, longitude, and LCC coordinates by subtracting the transformed coordinates from the control points' coordinates. The errors are computed as absolute differences for each coordinate pair (latitude, longitude, X, Y). Additionally, the script calculates the root mean square error (RMSE) to assess overall accuracy. If any error exceeds predefined thresholds, the user is prompted to adjust the transformation parameters (e.g., standard parallels in LCC projection) to optimize the results. This step ensures maximum accuracy and provides a reliable basis for mapping and flight route planning.

- **Scatter Map Validation:**

In addition to the above methods, I also used a scatter map in the code file to visually validate the transformed coordinates. By observing the points and their relationships on the map, it is possible to judge whether the transformed coordinates are correct.

(3) *Visualization:*

After applying the transformations, the results are visualized to assess their quality. The Python script uses matplotlib to display the C12-C8-C5 flight routes under both Mercator and Lambert Conformal Conic projections, comparing the differences between transformations based on 2 control points and 4 control points. This further deepens the analysis of how these factors affect the accuracy of the flight routes.

3.3 Quality Assurance: Least Squares and Validation

Least Squares Adjustment minimizes errors by optimizing transformation parameters using all control points, improving the transformation's accuracy. Residual analysis checks the difference between transformed coordinates and control points, ensuring the transformation's accuracy and reliability.

4. Results

4.1 Transformation Outputs

The transformation from Mercator coordinates to geographic coordinates, and then to LCC projection coordinates, resulted in residual errors of less than 12 km. This shows a high level of accuracy, especially given that the Mercator projection, known for its distortions at higher latitudes, was used as the starting point. The low residual errors indicate that the transformation minimized these distortions, with the LCC projection offering a more accurate representation of Madagascar's coastline.

- **Mercator Conversion**:

The Mercator to geographic coordinate transformation was highly precise, as shown by the minimal residual errors (less than 12 km). This confirms that the geographic coordinates closely match the expected values, preserving the accuracy of the coastline's location despite the Mercator projection's limitations at higher latitudes.

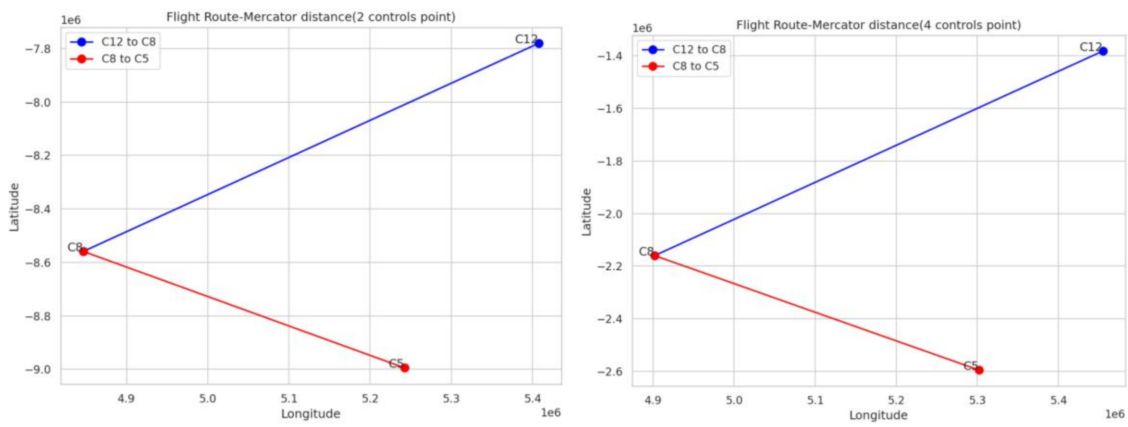

Figure 8 Flight Route (Madagascar Projection)

- **LCC Projection**:

The LCC coordinates for Madagascar's coastline accurately depict its features with minimal distortion. This confirms that the LCC projection, optimized for Madagascar's mid-latitude region (10°S to 25°S), provides a more accurate representation, especially in shape and scale. The transformation from geographic to LCC coordinates successfully improved mapping accuracy and minimized the distortions common in projections like Mercator.

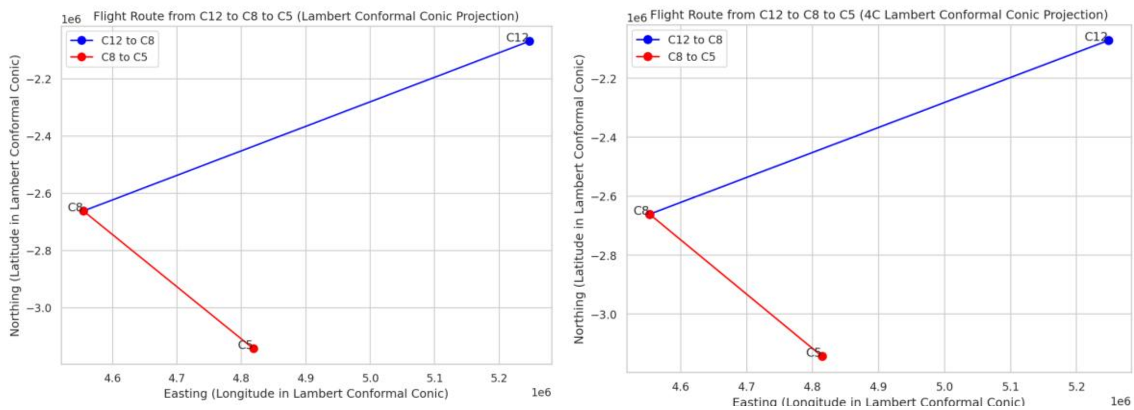

Figure 9 Flight Route (Lambert Conformal Conic Projection)

4.2 Visualization of Projection Differences

The visual comparison of the coastline in Mercator and LCC projections clearly highlights the differences in accuracy and distortion.

- **Mercator Projection**:

The coastline plotted in the Mercator projection shows significant distortion, particularly towards the southern parts of the island. This is consistent with the known behavior of the Mercator projection at higher latitudes, where areas and distances are exaggerated. The southern parts of Madagascar (closer to **25°S**) appear disproportionately stretched, leading to a less accurate representation of the coastline.

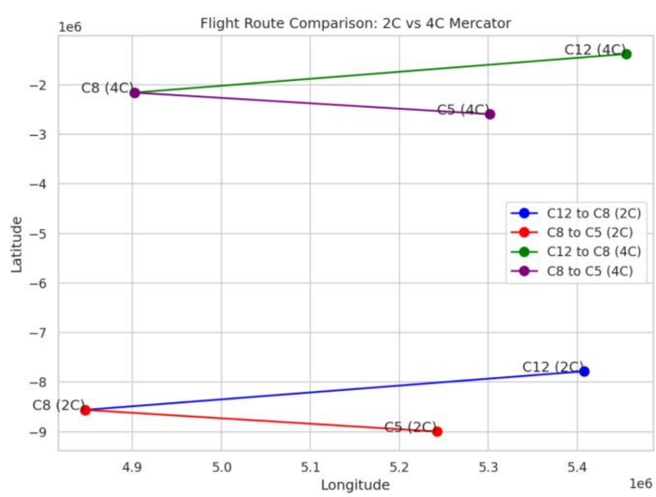

Figure10 Flight Route Comparison: 2C vs 4C Mercator

- **LCC Projection**:

In contrast, the LCC projection provides a more accurate representation of Madagascar's coastline, with reduced distortion across the region. The island's shape and scale are much more accurate, particularly in the mid-latitudes. The LCC projection effectively reduces the distortion seen in the Mercator projection, especially for regions like Madagascar, located between the Tropic of Capricorn and the Equator.

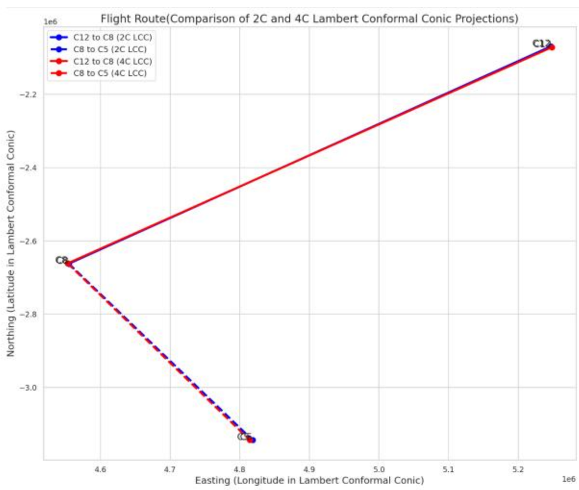

Figure11 Flight Route Comparison: 2C vs 4C LCC

- **Validation of Improved Accuracy**:

The comparison of the two projections highlights the significant improvement in the accuracy of the coastline representation using the LCC projection. The reduced distortion in the LCC projection allows for more precise mapping.

- **Cost Result:**

	name	time(h)	fuel_consumption (gallons)	fuel_consumption (liters)	fuel_cost (USD)	Total Distance (km)
0	Geodetic	7.952306	214.712255	812.773917	427.921524	1471.176562
1	Mercator(2 control points)	8.358254	225.672869	854.264335	449.766028	1546.277066
2	Mercator(4 control points)	8.358254	225.672869	854.264335	449.766028	1546.277066
3	Lambert Conformal Conic(2 control points)	7.895205	213.170545	806.937915	424.848897	1460.612997
4	Lambert Conformal Conic(4 control points)	7.894834	213.160523	806.899974	424.828921	1460.544321

Figure12 Flight Route Comparison: 2C vs 4C LCC

4.3 Visualization of Projection Differences

Figures comparing the coastline in Mercator and LCC projections validate improved accuracy.

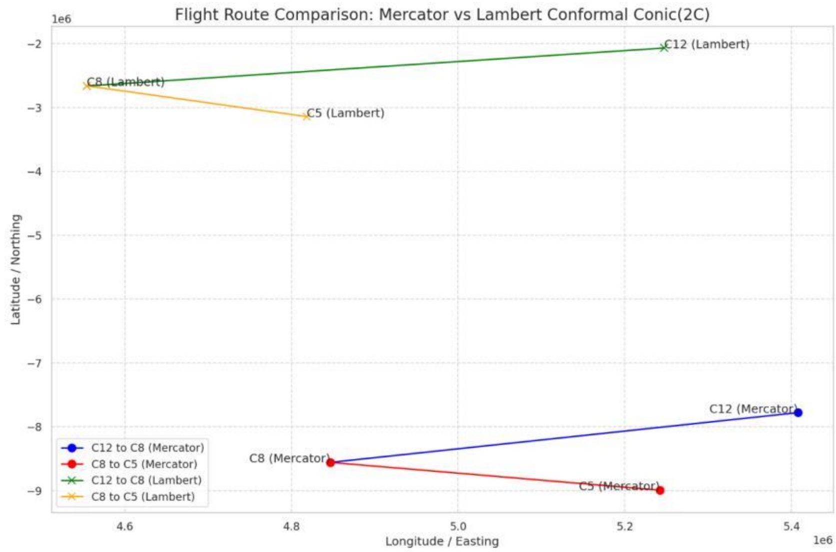

Figure13 Flight Route Comparison: Mercator vs Lambert Conformal Conic(2C)

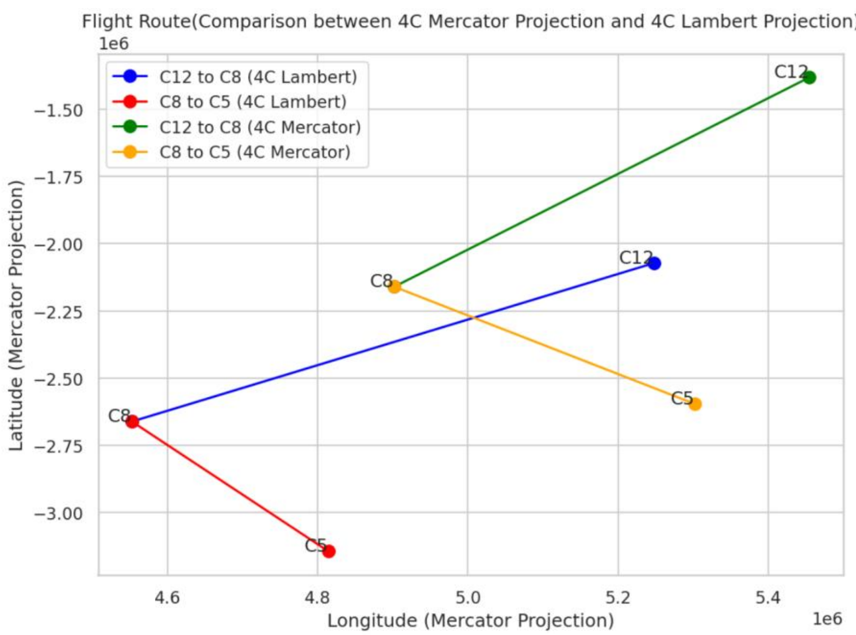

Figure14 Flight Route Comparison: Mercator vs Lambert Conformal Conic(4C)

5. Discussion

5.1 Projection Characteristics: A Comparative Analysis

Based on the analysis of the flight route comparison figure(*Figure14*), the Mercator projection introduces noticeable distortions in the C12-C8-C5 flight route, particularly stretching the C12-C8 segment and causing angular deviation in the C8-C5 segment. These distortions affect direction and distance accuracy, especially in latitude representation. In contrast, the LCC projection provides shorter, more direct flight paths, minimizing distortions and improving precision. This makes the LCC projection more suitable for mid-latitude regions like Madagascar, where accuracy in aviation mapping and fuel efficiency is essential.

5.3 Economic Implications for Lillybank Tours

Based on the cost analysis, the Mercator projection (2 and 4 control points) has the highest fuel cost of $449.77, while the LCC projection with 4 control points achieves the lowest cost of $424.83. This results in a cost difference of $24.94. The longer distance under the Mercator projection (1546.28 km) compared to the LCC projection (1460.54 km) accounts for this additional fuel consumption and higher cost, highlighting the advantage of LCC in minimizing distortions and improving efficiency.

6. Conclusion and Recommendations

Based on the comparison of flight routes in the Mercator and LCC projections, the Mercator projection shows noticeable directional deviations. As seen in the flight route map, the Mercator projection alters the angular relationships between points, particularly in the C12-C8 and C8-C5 segments, causing the routes to appear stretched and less direct. These deviations are a result of Mercator's inability to preserve shape and scale over regional distances, especially near mid-latitude regions like Madagascar.

In contrast, the LCC projection preserves the shape and relative distances more effectively. This leads to shorter and more direct flight routes, contributing to a reduction of 85.74 km in distance and $24.94 in fuel cost compared to the Mercator projection.

Future work could refine the placement of control points and explore dynamic adjustments to projection parameters to further improve route accuracy and operational efficiency.

7. References

1. Snyder, J. P., *Map Projections: A Working Manual*, U.S. Geological Survey Professional Paper 1395, 1987.
2. Python Libraries: pyproj, matplotlib.
3. Bell Long Ranger Helicopter, Technical Specifications.

8. Appendices

Appendix A: GEOG5008_Madagascar_2964649T_v4.ipynb
Appendix B: GEOG5008_Madagascar_2964649T_v4.html
Appendix C: GEOG5008_Madagascar_2964649T_Projection coordinate_and_cost_data.xlsx
Appendix D: GEOG5008_Madagascar_2964649T_Variable_Explanations.pdf